Dynamic Characteristics of a Damaged Plate

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It is very important to well understand the dynamic characteristics of damaged structures to successfully develop or to choose a most appropriate structural damage identification method (SDIM) as the means of non-destructive testing. In this paper, the dynamic equation of motion for damaged plates is derived by introducing a damage distribution function, which may characterize the effective state of structural damages. It is found that structural damages may induce the coupling between modal coordinates. The effects of damages on the vibration characteristics of a plate depending on their locations, sizes, and magnitudes are numerically investigated in a systematic way. The numerical investigations are also given to the effects of damage-induced modal coupling on the changes in vibration characteristics and to the minimum number of natural modes required to predict sufficiently accurate vibration characteristics of damaged plates.

Key Words: Structural Damages, Plates, Vibration Response, Natural Frequency, Damage Influence Matrix, Damage-Induced Modal Coupling

1. Introduction

Structural damages may lead to the changes in dynamic characteristics of a structure such as the vibration response, natural frequency, mode shape, and the modal damping. The changes in dynamic characteristics of the structure can be used in turn to detect, locate and quantify the structural damages generated within the structure (Doebling et al., 1998). Thus, it is very important to well understand the dynamic characteristics of damaged structures to successfully develop or to choose an appropriate SDIM as the means of non-destructive testing.

In recent years, there have been many studies on the vibration analysis of cracked one-dimensional structures. The extensive review can be found in the article by Dimarogonas (1996). The natural frequency changes of beams due to cracks, notches, or other geometrical changes were investigated analytically by Gudmundson (1982) and Sato (1983), and experimentally by Divini *et al.* (1995). For structural damage identifications, the changes in natural frequencies, mode shapes, and curvature mode shapes due to the existence of damages (depending on their locations and dimensions) have been investigated by other researchers (e. g., Weissenburger, 1968; Yuen, 1985; Pandey et al., 1991; Davini et al., 1995; Banks et al., 1996; Luo and Hanagud, 1997; Lee at al., 2000). Most of previous studies have been confined to the structures such as beams, truss structures, and frame structures.

The analytical approach for the vibrations of cracked rectangular plates was first carried out by Lynn and Kumbasar (1967) and then advanced by Stahl and Keer (1972). The numerical study has been suggested by Chen (1984) using hybrid-displacement FEM and by Leung and Su (1996) using a fractal two level FEM. With no emphasis on crack detection application, Lee and Lim (1993) investigated the vibration of rectangular plate with a centrally located crack. Very recently,

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Khadem and Rezaee (2000) introduced an analytical approach to observe the vibration of the rectangular plate with all-over part-through crack. In summary, to the authors' best knowledge, the major concern in the previous studies on cracked or damaged plates has been in the development of solution methods or to investigate vibration characteristics of the plates with very specific cracks.

The purpose of the present paper is to investigate the effects of structural damages on the dynamic characteristics of damaged plates depending on their locations and magnitudes. The effects of damage-induced modal coupling as well as the effects of the modal parameters of intact plate on the accuracy of the predicted dynamic characteristics of damaged plates are also investigated.

2. Dynamics of Damaged Plates

2.1 Dynamic equation of motion for damaged plates

Consider an elastic, isotropic, and thin rectangular plate with the width a, the length b, and the thickness h. For small amplitude vibration, the dynamic equation of motion for the plate is given by (Yu, 1996)

$$\frac{\partial^2 M_x}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial^2 M_y}{\partial y^2}+f(x, y, t)=m\ddot{w} \quad (1)$$

where $w(x, y, t)$ is the flexural deflection, $f(x, y, t)$ the external force applied normal to the surface of plate, m is the mass density per area, and dot ($\cdot$) denotes the partial derivative with respect to time t. The moment resultants M_x, M_{xy}, and M_y are defined, respectively, by

$$M_x=-K\left[\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right],\ M_y=-K\left[\nu\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right],$$
$$M_{xy}=-K(1-\nu)\frac{\partial^2 w}{\partial x\partial y} \quad (2)$$

where K is the flexural rigidity for intact plates, which is given by

$$K=\frac{Eh^3}{12(1-\nu^2)} \quad (3)$$

where E and ν are Young's modulus and Poisson's ratio for intact material, respectively.

For most practical problems, it would be difficult to assign a definitive representation for the stiffness of a damaged area because the location, dimensions, and geometry of the damaged site are unknown in prior. Thus, one of the simplest approaches is to represent the damage-induced change in stiffness by the degradation of the elastic modulus at damage locations (Yuen, 1985; Banks et al. , 1996; Luo and Hanagud, 1997).

$$\bar{E}(x, y)=E[1-d(x, y)] \quad (4)$$

where $\bar{E}$ is the effective Young's modulus for damaged material, and $d(x, y)$ is the damage distribution function which may implicitly characterize the states of local damages. The case $d(x, y)=0$ indicates the intact state, while $d(x, y)=1$ indicates the complete ruptures of material at damage locations. Using Eq. (4), the flexural rigidity for damaged plates can be expressed in the form

$$\bar{K}(x, y)=K[1-d(x, y)] \quad (5)$$

It will be reasonable to assume that structural damages do not change the mass distribution because they will result in stiffness losses instead of complete breakage with a loss of mass (Yuen, 1985; Pandey et al, 1991). Thus, replacing the intact flexural rigidity K in Eq. (2) with the value for damaged state $\bar{K}$ of Eq. (5) and substituting Eq. (2) into Eq. (1) may yield a governing equation of motion for damaged plates as follows:

$$K\nabla^4 w-K\left\{\frac{\partial^2}{\partial x^2}\left[d\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right)\right]+2\frac{\partial^2}{\partial x\partial y}\left[d(1-\nu)\frac{\partial^2 w}{\partial x\partial y}\right]\right.$$
$$\left.+\frac{\partial^2}{\partial y^2}\left[d\left(\nu\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)\right]\right\}+m\ddot{w}=f(x, y, t) \quad (6)$$

where ∇^4 denotes the biharmonic operator. For intact plates, the second term in the left side of Eq. (6) vanishes. In this study, it is assumed that there are no damages along the boundaries of plate. Thus, the boundary conditions for intact plates can be equally applied to damaged plates.

2.2 Dynamic responses of intact plates

By putting $d=0$ in Eq. (6), the dynamic equation of motion for intact plates is obtained as

$$K\nabla^4 w+m\ddot{w}=f(x, y, t) \quad (7)$$

The forced vibration response of an intact plate can be obtained by superposing its natural modes as

$$w(x, y, t)=\sum_m \sum_n W_{mn}(x, y) q_{mn}(t) \equiv \sum_\alpha W_\alpha(x, y) q_\alpha(t) \quad (8)$$

where W_{mn} (or W_α) and q_{mn} (or q_α) are the natural modes and modal (or generalized) coordinates for the intact plate. As shown in the far right side of Eq. (8), the contracted subscripts for natural mode numbers will be consistently used in the following for brevity: *i. e.*, α for mn and β for *rs*.

The natural modes W_α should satisfy the eigenvalue problem for intact plate as

$$K\nabla^4 W_\alpha = m\Omega_\alpha^2 W_\alpha \quad (9)$$

and the orthogonality property

$$\int\int m W_\alpha W_\beta dxdy = \delta_{\alpha\beta} \quad (10)$$

$$\int\int K W_\beta \nabla^4 W_\alpha dxdy = \Omega_\alpha^2 \delta_{\alpha\beta} \quad (11)$$

where Ω_α are the natural frequencies of intact plate and $\delta_{\alpha\beta}$ is the Kronecker delta.

Substituting Eq. (8) into Eq. (7) and applying the orthogonality property for natural modes yields the modal equations as

$$\ddot{q}_\alpha + \Omega_\alpha^2 q_\alpha = f_\alpha(t) \quad (12)$$

where f_α are modal (or generalized) forces defined by

$$f_\alpha(t) = \int\int f(x, y, t) W_\alpha dxdy \quad (13)$$

Assume that a harmonic point force is applied at a specified point (x_F, y_F) as

$$f(x, y, t) = F(x, y) e^{i\omega t} = F_0 \delta(x-x_F) \delta(y-y_F) e^{i\omega t} \quad (14)$$

where F_0 and ω are the amplitude and (circular) frequency of the excitation force, respectively. Substituting Eq. (14) into Eq. (13) gives the modal forces in the form

$$f_\alpha(t) = W_\alpha(x_F, y_F) F_0 e^{i\omega t} \quad (15)$$

From Eqs. (12) and (15), the modal coordinates can be obtained as

$$q_\alpha(t) = \frac{W_\alpha(x_F, y_F)}{\Omega_\alpha^2 - \omega^2} F_0 e^{i\omega t} \equiv Q_\alpha e^{i\omega t} \quad (16)$$

The forced vibration response for an intact plate can be obtained by substituting Eq. (16) into Eq. (8).

2.3 Dynamic responses of damaged plates

Dynamic equation of motion for damaged plates is given by Eq. (6). Assume that the same harmonic point force that is applied to a plate of intact state is consistently applied to the plate of damaged state. The forced vibration response of a damaged plate can be obtained by superposing the natural modes for its intact state as (Dowell, 1975; Meirovitch, 1980):

$$w(x, y, t) = \sum_\alpha W_\alpha(x, y) \bar{q}_\alpha(t) \quad (17)$$

where $\bar{q}_\alpha$ are the modal coordinates for the damaged plate. Substituting Eq. (17) into Eq. (6) and applying the orthogonality property for natural modes yields the modal equations as

$$\ddot{\bar{q}}_\alpha + \Omega_\alpha^2 \bar{q}_\alpha - \sum_\beta \lambda_{\alpha\beta} \bar{q}_\beta = f_\alpha(t) \quad (18)$$

where

$$\lambda_{\alpha\beta} = K\int\int \left\{ \frac{\partial^2}{\partial x^2}\left[d\left(\frac{\partial^2 W_\alpha}{\partial x^2} + \nu \frac{\partial^2 W_\alpha}{\partial y^2} \right) \right] + 2\frac{\partial^2}{\partial x \partial y}\left[d(1-\nu) \frac{\partial^2 W_\alpha}{\partial x \partial y} \right] + \frac{\partial^2}{\partial y^2}\left[d\left(\nu \frac{\partial^2 W_\alpha}{\partial x^2} + \frac{\partial^2 W_\alpha}{\partial y^2} \right) \right] \right\} W_\beta dxdy \quad (19)$$

Equation (18) says that the natural frequencies of damaged plate, $\bar{\Omega}_\alpha$, can be obtained from

$$\det[(\Omega_\alpha^2 - \bar{\Omega}_\alpha^2)\delta_{\alpha\beta} - \lambda_{\alpha\beta}] = 0 \quad (20)$$

The last term in the left side of Eq. (18) reflects the influence of damage, which is characterized by the matrix $\lambda_{\alpha\beta}$. Applying the integration by parts for two-dimensional problem into Eq. (19) and assuming that there are no damages along the boundaries, the matrix $\lambda_{\alpha\beta}$ can be expressed as

$$\lambda_{\alpha\beta} = \int\int d(x, y) \Phi_{\alpha\beta}(x, y) dxdy \quad (21)$$

where

$$\Phi_{\alpha\beta}(x, y) = K\left[\left(\frac{\partial^2 W_\alpha}{\partial x^2} + \nu \frac{\partial^2 W_\alpha}{\partial y^2} \right) \frac{\partial^2 W_\beta}{\partial x^2} + 2(1-\nu) \frac{\partial^2 W_\alpha}{\partial x \partial y} \frac{\partial^2 W_\beta}{\partial x \partial y} + \left(\nu \frac{\partial^2 W_\alpha}{\partial x^2} + \frac{\partial^2 W_\alpha}{\partial y^2} \right) \frac{\partial^2 W_\beta}{\partial y^2} \right] \quad (22)$$

The matrix $\lambda_{\alpha\beta}$, called herein 'damage influence matrix (DIM)', is symmetric and depends on the mode shape curvatures and damage distribution function $d(x, y)$. It can be observed from Eq. (18) that the off-diagonal terms of DIM induce the coupling between modal coordinates, which is called herein 'damage-induced modal coupling (DIMC)'. To the authors' best knowledge, the DIMC has been neither introduced nor discussed in the previous literature.

Assume the general solutions of Eq. (18) in the form

$$\bar{q}_\alpha(t) = q_\alpha(t) + \Delta q_\alpha(t) \tag{23}$$

where q_α are the modal coordinates for intact plate and Δq_α are the small perturbations of q_α due to the presence of damages. Substituting Eq. (23) into Eq. (18) yields

$$\Delta\ddot{q}_\alpha + \Omega_\alpha^2 \Delta q_\alpha - \sum_\beta \lambda_{\alpha\beta}\Delta q_\beta \cong \sum_\beta \lambda_{\alpha\beta} Q_\beta e^{i\omega t} \tag{24}$$

where Q_β are given by Eq. (16). Equation (24) can be solved for Δq_α to obtain

$$\Delta q_\alpha(t) = \sum_\beta \sum_\gamma [(\Omega_\alpha^2 - \omega^2)\delta_{\alpha\gamma} - \lambda_{\alpha\gamma}]^{-1} \lambda_{\gamma\beta} Q_\beta e^{i\omega t} \tag{25}$$

Because the third term $\Sigma \lambda_{\alpha\beta}\Delta q_\beta$ in the left side of Eq. (24) is very small when compared with the other terms, it can be neglected to obtain the approximation as

$$\Delta q_\alpha(t) = \sum_\beta \frac{\lambda_{\alpha\beta} Q_\beta}{\Omega_\alpha^2 - \omega^2} e^{i\omega t} \tag{26}$$

Substituting Eqs. (16) and (26) into Eq. (23) and its result into Eq. (17) yields the forced vibration response for the damaged plate as

$$w(x, y, t) = \left[\sum_\alpha \frac{W_\alpha(x, y)\, W_\alpha(x_F, y_F)}{\Omega_\alpha^2 - \omega^2} + \sum_\alpha \sum_\beta \lambda_{\alpha\beta} \frac{W_\alpha(x, y)}{\Omega_\alpha^2 - \omega^2} \frac{W_\beta(x_F, y_F)}{\Omega_\beta^2 - \omega^2} \right] F_0 e^{i\omega t} = W(x, y) e^{i\omega t} \tag{27}$$

The structural damping can be readily taken into account in the previous formulations, if needed, by simply replacing the flexural rigidity K and natural frequencies Ω_α with $K(1+i\eta)$ and $\Omega_\alpha(1+i\eta)^{1/2}$, respectively: where η is the structural damping factor (or loss factor) (Meirovitch, 1980). One may use the 'equivalent' loss factor for the case of non-structural damping. However, the damping will not be considered in the present study, for brevity, in order to focus our discussions on the effects of damage only.

2.4 Damage influence matrix

Equation (21) shows that DIM depends on both mode shapes and damage distribution over the plate. Though the present discussion can be readily extended to any damage distribution over the plate, we assume here that damages are uniform through the thickness of plate and distributed uniformly over small local areas, *i. e.*, the piece-wise uniform thickness-through damages. Figure 1 shows a piece-wise uniform thickness-through damage over a small area $4\bar{x}\bar{y}$, centered at (x_d, y_d). The piece-wise uniform damage can be represented by

$$d(x, y) = D[H(x_d - \bar{x}) - H(x_d + \bar{x})][H(y_d - \bar{y}) - H(y_d + \bar{y})] \tag{28}$$

where $0 \le D \le 1$ is the damage magnitude uniformly distributed over the area of $4\bar{x}\bar{y}$, and $H(z)$ is the Heaviside's unit function defined by

$$H(z) = \begin{cases} 1 & \text{when } z > 0 \\ 0 & \text{when } z < 0 \end{cases} \tag{29}$$

Substituting Eq. (28) into Eq. (21) gives the DIM for the plate with a single damage at (x_d, y_d) as follows:

$$\lambda_{\alpha\beta} = k_{\alpha\beta} D \tag{30}$$

where

$$k_{\alpha\beta} = \int_{y_d - \bar{y}}^{y_d + \bar{y}} \int_{x_d - \bar{x}}^{x_d + \bar{x}} \Phi_{\alpha\beta}(x, y)\, dx dy \tag{31}$$

Equations (30) and (31) can be further generalized for the plate with N damages as follows:

$$\lambda_{\alpha\beta} = \sum_{j=1}^{N} k_{\alpha\beta}^j D_j \quad (j = 1, 2, \cdots, N) \tag{32}$$

where

$$k_{\alpha\beta}^j = \int_{y_{dj} - \bar{y}_j}^{y_{dj} + \bar{y}_j} \int_{x_{dj} - \bar{x}_j}^{x_{dj} + \bar{x}_j} \Phi_{\alpha\beta}(x, y)\, dx dy \tag{33}$$

where D_j, (x_{dj}, y_{dj}), $2\bar{x}_j$, and $2\bar{y}_j$ represent the magnitude, location, and the dimensions of the j-th local damage, respectively.

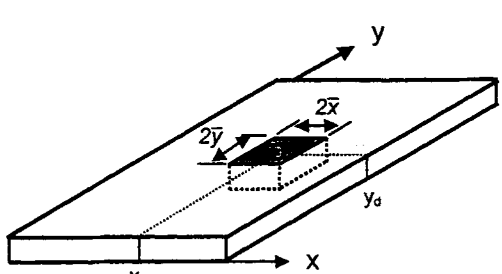

Fig. 1 A piece-wise uniform thickness-through damage

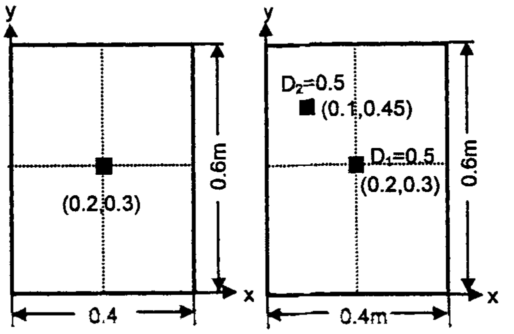

Fig. 2 Illustrative problems : the simply supported rectangular plates with (a) a single damage, and (b) two damages

3. Numerical Illustrations and Discussions

Equations (20) and (27) show that damage-induced changes in natural frequencies and vibration response of a plate mainly depend on damage characteristics such as the locations (x_{dj}, y_{dj}), dimensions $(2\bar{x}_j, 2\bar{y}_j)$ and magnitudes (D_j) of damages. Equations (20) and (27) also show that the accuracy of the predicted natural frequencies and vibration response of a damaged plate depends on how many modal parameters (*i. e.*, natural modes and natural frequencies) of the intact plate are taken into account in the analysis and on whether the DIMC (*i. e.*, the off-diagonal terms of DIM) is neglected or not. To numerically investigate above issues, the simply supported rectangular plates with one and two identical piece-wise uniform damages, as shown in Fig. 2, are considered as the illustrative problems. The plates have the thickness 0.4 cm and the dimensions 0.4 *m* and 0.6 *m* in the x- and y-directions, respectively. The material properties of the plates are the intact Young's modulus E= 72 *GPa*, Poisson's ratio ν=0.3, and the mass density 2800 kg/m^3.

The DIM for the plate with a single damage is given in Table 1. Similarly, the DIM for the plate with two damages is given in Table 2. Tables 1 and 2 show that, as a general rule, the diagonal terms of DIM (*i.e.*, direct effects of damages) increase in magnitude as the mode number

Table 1 Non-dimensionalized damage influence matrix $(\lambda_{mnrs}/\lambda_{1111})$ for the simply-supported rectangular plate with a single damage : D=0.5 ; x_D=0.2m : y_D=0.3m ; $2\bar{x}=2\bar{y}$=0.02m ; λ_{1111}=193.25

(r,s) \ (m,n)	(1 1)	(2 2)	(3 3)	(4 4)	(5 5)	···	(9 9)
(1 1)	1.0	0.00	−2.84	0.00	0.00	···	8.89
(2 2)		0.02	0.00	0.00	0.00	···	0.00
(3 3)			13.80	0.00	0.00	···	−25.23
(4 4)				0.10	0.00	···	0.00
(5 5)	$\frac{\lambda_{mnrs}}{\lambda_{1111}}$				26.33	···	0.00
⋮	SYMMETRIC					···	⋮
(9 9)							79.11

Table 2 Non-dimensionalized damage influence matrix $(\lambda_{mnrs}/\lambda_{1111})$ for the simply-supported rectangular plate with two damages : $D_1=D_2$=0.5 ; x_{D1}=0.2m, y_{D1}=0.3m, x_{D2}=0.1m, y_{D2}=0.4m ; $2\bar{x}_1=2\bar{x}_2=2\bar{y}_1=2\bar{y}_2$=0.02$m$; λ_{1111}=275.15

(r,s) \ (m,n)	(1 1)	(2 2)	(3 3)	(4 4)	(5 5)	···	(9 9)
(1 1)	1.0	0.47	−2.43	1.39	0.95	···	5.58
(2 2)		0.67	−1.25	1.14	0.33	···	0.23
(3 3)			9.87	−2.25	−2.36	···	−15.57
(4 4)				3.90	1.99	···	3.10
(5 5)	$\frac{\lambda_{mnrs}}{\lambda_{1111}}$				15.88	···	6.22
⋮	SYMMETRIC					···	⋮
(9 9)							48.79

increases. However, they decrease momentary at a certain vibration mode if the nodes of the mode are very near to or inside of damaged small zones. For instance, the values of λ_{2222} and λ_{4444} in Table 1 are decreased because a node of the (22, 22) and (44, 44) modes coincide with damage location. Equation (32) shows that, in general, DIM becomes larger as the magnitudes of damages increase. The off-diagonal terms of DIM, which represent the DIMC or the indirect effects of damage, are relatively small when compared with the diagonal terms. As can be proved from Eq. (21) by using the orthogonality property for natural modes, the off-diagonal terms vanish com-

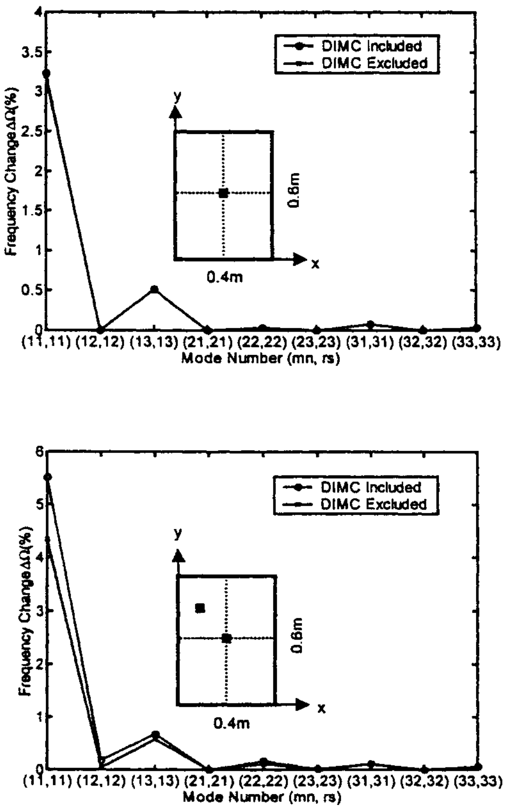

Fig. 3 Damage-induced modal coupling (DIMC) dependence of the changes in natural frequencies of the simply supported plate with (a) a single damage and (2) two damages

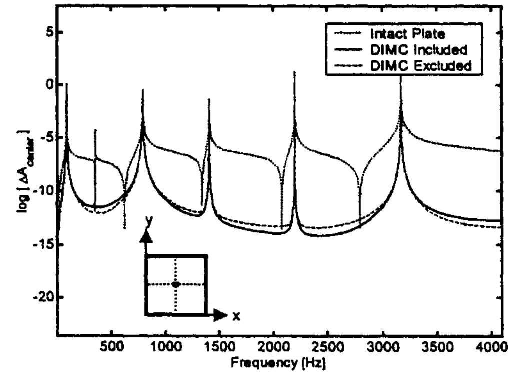

Fig. 4 Effects of damage-induced modal coupling (DIMC) on the 'inertance' FRF of the simply supported plate with a single damage.

pletely when damage is uniformly distributed over the whole plate, regardless of its magnitude.

Figure 3 shows the effects of DIMC on the damage-induced (percent) changes in natural frequencies of plate. Neglecting DIMC seems to underestimate the damage-induced changes in natural frequencies. In general, the effects of DIMC on the changes in natural frequencies are found to be negligible, especially when the damage is very weak. However, it is desirable to include the DIMC in the analysis because damages are not known in prior for most practical cases. Figure 3 shows that in general the damage-induced changes in natural frequencies at the lower modes are larger than those at the higher modes. The damage-induced changes in natural frequencies become negligible as the mode number increases and highly depend on mode number and damage location. If damages are located at the nodes of a mode, the damage-induced changes in the natural frequency of the corresponding mode become very small. For instance, the damage-induced changes in natural frequencies are very small for (12, 12) and (21, 21) modes. Figure 4 compares the changes in the inertance FRF depending on whether the DIMC is included or not in the computation of FRF. The change in the inertance FRF, ΔA_{center}, is defined as the difference between the inertance FRF of damaged plate from that of intact plate, all measured at the *center* of plate. In general, the damage tends to reduce the amplitude of FRF, but the effects of DIMC on the changes in FRFs are very small.

Though a sufficiently large number of modal parameters of intact plate are required for accurate prediction of the effects of damage, only a limited number of the lower modal parameters are available from the modal testing or the theoretical modal analysis. Thus, the errors due to the omitted (higher) modes will be inevitable. Figure 5 shows the ratios of the omitted modes-induced errors in natural frequencies to the damage-induced changes in natural frequencies for the simply supported plate with a single damage. The omitted modes-induced errors in natural frequencies, denoted by $\Delta\Omega$(omitted modes) in the figure, are defined by the differences of the exact

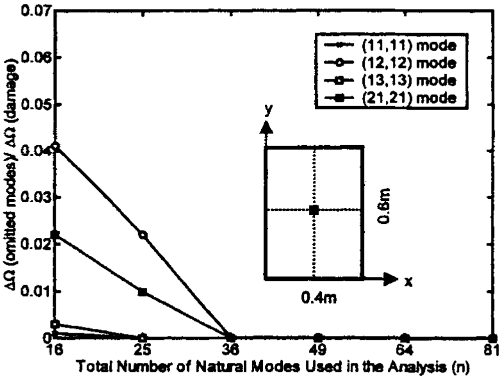

Fig. 5 The ratios of the omitted modes-induced errors in natural frequencies to the damage-induced changes in natural frequencies for the simply supported plate with a single damage.

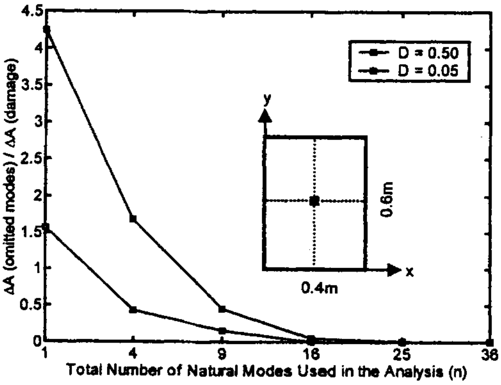

Fig. 6 The ratios of the omitted modes-induced errors in inertance FRFs to the damage-induced changes in inertance FRFs for two different damage magnitudes.

natural frequencies of damaged plate from the approximate ones calculated by using a finite number of natural modes. On the other hand, the damage-induced changes in natural frequencies, denoted by $\Delta\Omega$(damage) in the figures, are defined by the differences of the exact natural frequencies of intact plate from those of damaged plate. The exact natural frequencies of intact plate are readily available from Blevins (1979), wheras those for damaged plate are computed from Eq. (20) by taking into account a sufficient number of natural modes of intact plate: *i.e.*, 144 natural modes. It is important to remind that the omitted modes-induced errors should be much smaller than pure damage-induced changes for reliable damage identification. Because the damage considered for Fig. 5 is located at a node of the (12, 12) and (21, 21) modes, about 25 and 16 natural modes are required for $\bar{\Omega}_{1212}$ and $\bar{\Omega}_{2121}$, respectively, to lower the omitted modes-induced errors 2 % below the damage-induced errors. The numbers of natural modes required for $\bar{\Omega}_{1212}$ and $\bar{\Omega}_{2121}$ are much larger than required for the other natural frequencies of which modes have not a node at the damage location. This means that, if damages are located at the nodes of a mode, the omitted modes-induced errors for the natural frequency corresponding to the mode become very significant. Figure 6 shows the ratios of the omitted modes-induced errors in inertance FRFs to the damage-induced changes in inertance FRFs for two different damage magnitudes. The inertance FRFs are measured at the midpoint of plate by applying a harmonic point force of ω= 100 Hz at the same point. Figure 6 shows that about 16 and 25 natural modes should be considered when D=0.5 and D=0.05, respectively, to lower the omitted modes-induced errors 2 % below the pure damage-induced errors. In general, a larger number of natural modes will be required for weakly damaged plates to reduce the omitted modes-induced errors to a required level.

The damage-induced changes in natural frequencies depending on the magnitude and location of a single damage are shown in Figs. 7 and 8, respectively. As can be hinted from Eq. (20), Fig. 7 shows that the damage-induced changes in natural frequencies increase almost in proportion to damage magnitudes. It can be observed from Figs. 7 and 8 that the sensitivity of the natural frequency to damage location is relatively high at the lower modes and the damage-induced changes in natural frequencies become negligible at the higher modes. It can be also observed from Figs. 7 and 8 that the damage-induced changes in natural frequencies are very small when the damage is located at a node of the corresponding vibration mode.

The effects of damage magnitude and location on the vibration amplitude at the middle of plate

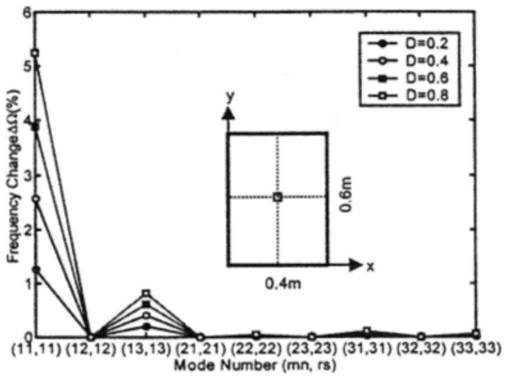

Fig. 7 Damage magnitude dependence of the changes in natural frequencies of damaged plate : $x_D=0.2m$; $y_D=0.3m$; $2\bar{x}=2\bar{y}=0.02m$

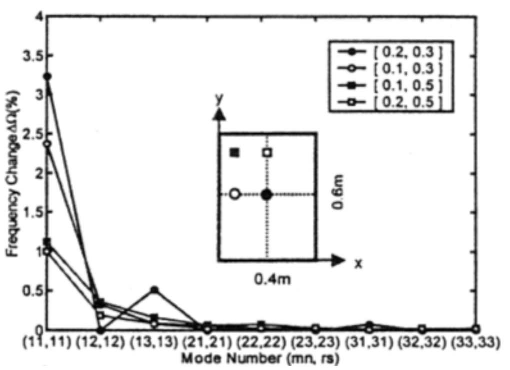

Fig. 8 Damage location dependence of the changes in natural frequencies of damaged plate : $D=0.5$; $2\bar{x}=2\bar{y}=0.02m$

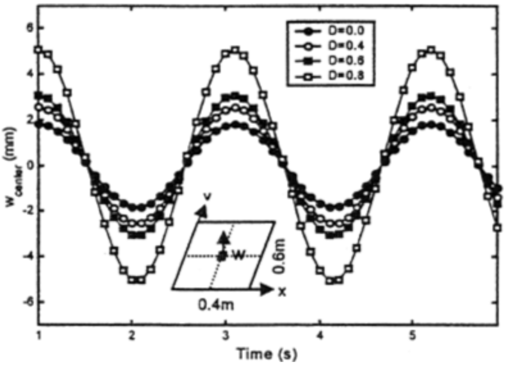

Fig. 9 Damage magnitude dependence of the displacement at the center of damaged plate : $x_D=0.2m$; $y_D=0.3m$; $2\bar{x}=2\bar{y}=0.02m$

are shown in Fig. 9 and Fig. 10, respectively. It is

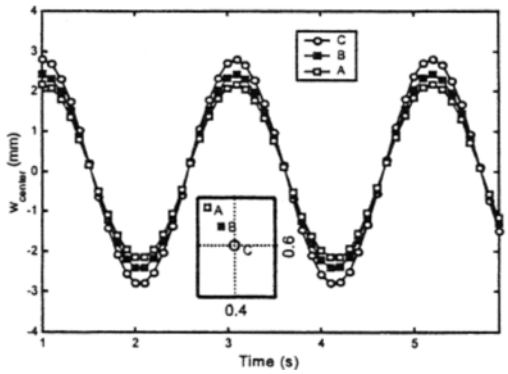

Fig. 10 Damage location dependence of the displacement at the center of damaged plate : $D=0.5$; $2\bar{x}=2\bar{y}=0.02m$

found that the vibration amplitude increases as the damage magnitude increases for the present example problem. Figure 10 shows that the effects of damage on the vibration amplitude become larger when the damage is located near the boundary rather than when it is located at the middle of plate.

4. Conclusions

In this study, the dynamic equation of motion is derived for damaged plates by introducing a damage distribution function to characterize the damage state. It is found that structural damages result in the coupling between modal coordinates. The effects of damages on the vibration characteristics of a plate depending on their locations, sizes, and magnitudes are numerically investigated. It is found that the damaged-induced changes in natural frequencies are much larger at the lower modes, but strongly dependent on damage locations. The numerical simulations show that the effects of damage-induced modal coupling on the changes in vibration characteristics are negligible for the example problems. It is also found that a sufficiently large number of natural modes should be used in the analysis to lower the omitted modes-induced errors in the vibration characteristics of the damaged plate far below the pure damage-induced errors.

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